Polar codes – a new paradigm in communication

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Overview of the talk

- Introduction: Transmission over channels
- Binary Polar Codes
- Nonbinary Polar Codes
- Improved decoding: Approaching optimal performance

Entropy

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Suppose $P_X = (P_X(x), x \in \mathcal{X})$ is a pmf

The expected number of bits is $H(X) = \sum_{x\in\mathcal{X}} P_X(x) \log \frac{1}{P_X(x)}$

$H(X)$ is called Shannon entropy

Transmission of information

Let $W : \mathcal{X} \to \mathcal{Y}$ be a stochastic mapping, $|\mathcal{Y}| < \infty$
$W(y|x) = \Pr(Y = y|X = x)$
Conditional entropy (residual uncertainty about X given Y)

$$H(X|Y) = E_{XY} \log \frac{1}{P_{X|Y}(x|y)}$$

Mutual information $I(X; Y) := H(X) - H(X|Y)$

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There exists a subset $D \subset \mathcal{X}^n$ such that its images in $\mathcal{Y}^n$ under W can be distinguished with high probability as long as $|D| < 2^{nI(W)}$.

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$I(W) = \max_{P_X} I(X; Y)$ is called Channel Capacity

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Mutual information $I(X; Y) := H(X) - H(X|Y) = \log q - H(X|Y)$

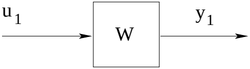
Theorem (Shannon '48)

There exists a subset $D \subset \mathcal{X}^n$ such that its images in $\mathcal{Y}^n$ under W can be distinguished with high probability as long as $|D| < 2^{nI(X;Y)}$.

$I(W) = \log q - H(X|Y)$ (uniform) Channel Capacity

Binary polar codes (Arikan '09)

Discrete memoryless channel $W : \mathcal{X} \to \mathcal{Y}$ with capacity $I(W)$



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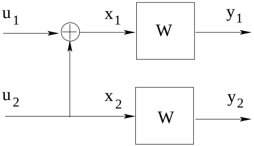

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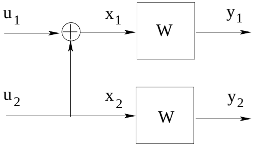

$$2I(W) = I(U_1 U_2; Y_1 Y_2) = I(U_1; Y_1 Y_2) + I(U_2; Y_1 Y_2 | U_1)$$
$$= I(U_1; Y_1 Y_2) + I(U_2; Y_1 Y_2 U_1)$$

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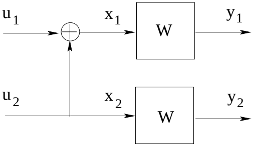

$$\begin{aligned}2I(W) &= I(U_1U_2;Y_1Y_2) = I(U_1;Y_1Y_2) + I(U_2;Y_1Y_2|U_1)\\ &= I(U_1;Y_1Y_2) + I(U_2;Y_1Y_2U_1)\end{aligned}$$

$$W^-(y_1y_2|u_1) = \frac{1}{2}\sum_{u_2=0}^{1} W(y_1|u_1\oplus u_2)W(y_2|u_2)$$

$$W^+(y_1y_2,u_1|u_2) = \frac{1}{2}W(y_1|u_1\oplus u_2)W(y_2|u_2).$$

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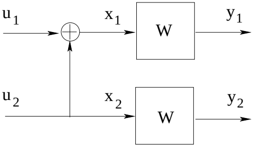

$$2I(W) = I(U_1; Y_1 Y_2) + I(U_2; Y_1 Y_2, U_1)$$

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$$W^+(y_1 y_2, u_1 | u_2) = \frac{1}{2} W(y_1 | u_1 \oplus u_2) W(y_2 | u_2)$$

$$I(W^+) \geq I(W) \geq I(W^-)$$

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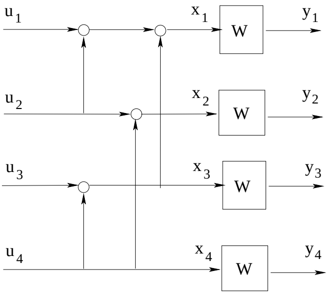

$$W^{++}, W^{+-}, W^{-+}, W^{--}$$

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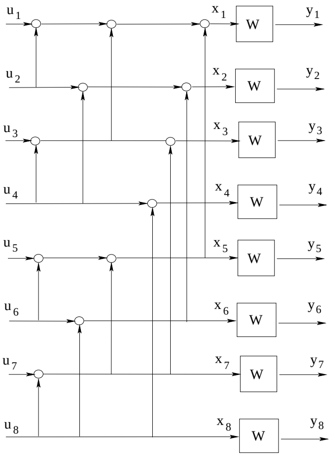

$$W^{+++}, W^{++-}, \ldots, W^{---}$$

Binary polar codes, BEC(0.5)

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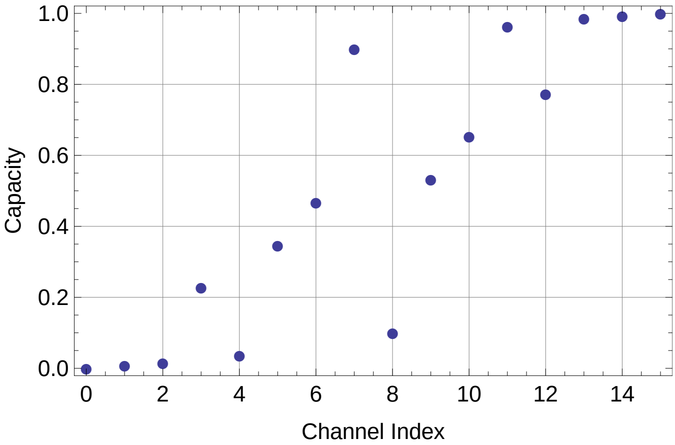
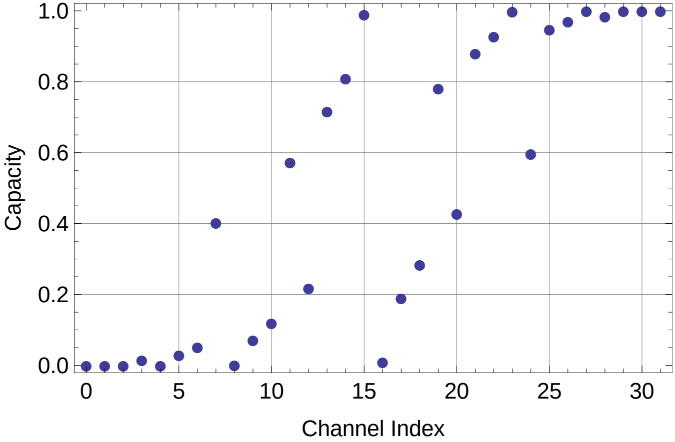
$$I(W^{--}) = 0.0625, I(W^{-+}) = 0.4375, I(W^{+-}) = 0.5625, I(W^{++}) = 0.9375$$

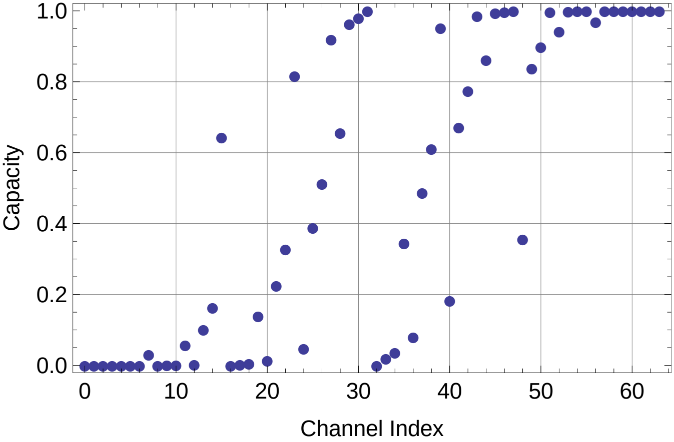
Polarization for BEC(0.5), $N = 16$



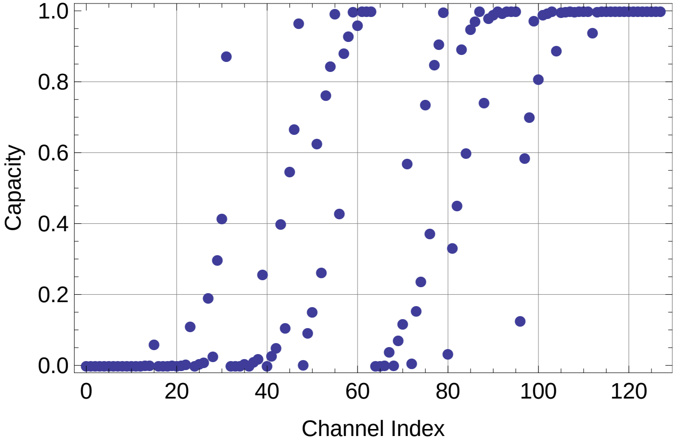
Polarization for BEC(0.5), $N = 32$



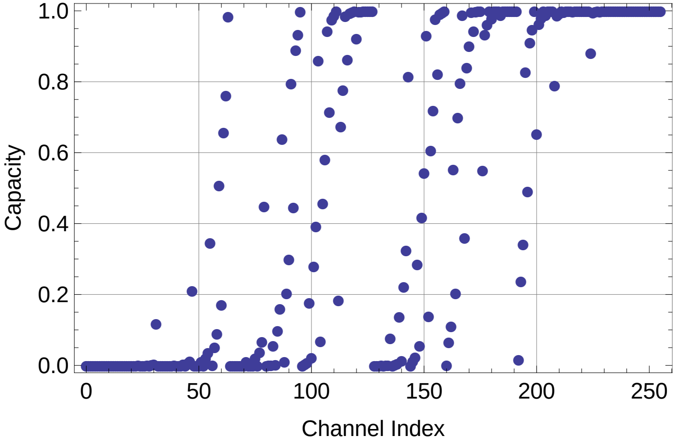
Polarization for BEC(0.5), $N = 64$



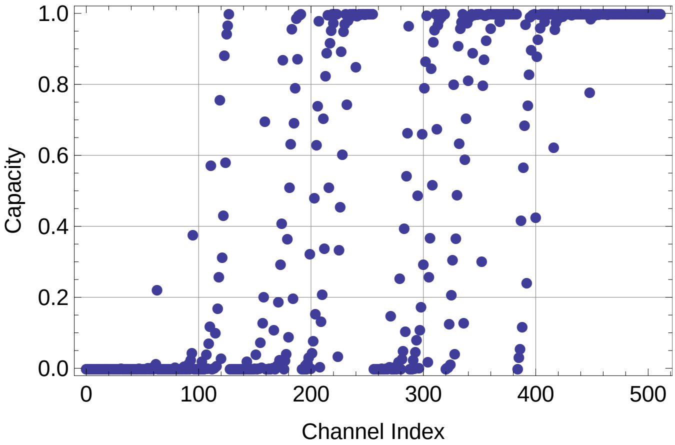
Polarization for BEC(0.5), $N = 128$



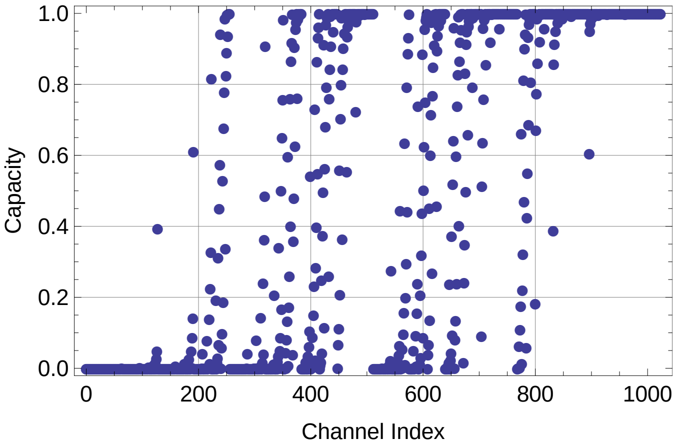
Polarization for BEC(0.5), $N = 256$



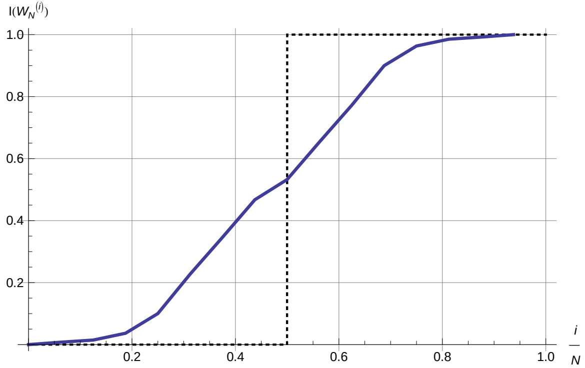
Polarization for BEC(0.5), $N = 512$



Polarization for BEC(0.5), $N = 1024$



Binary polar codes: ordered bits ($N = 2^4$)



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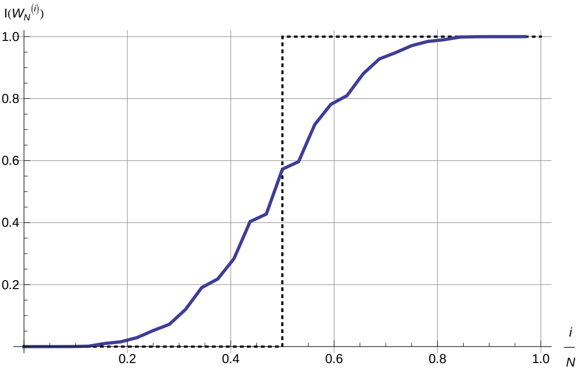

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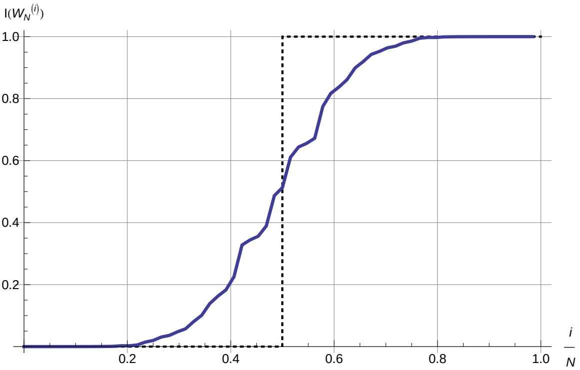

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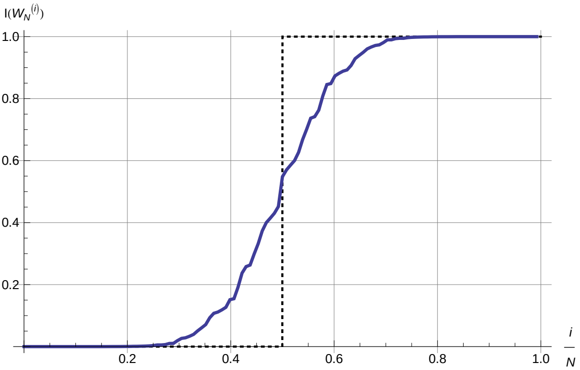

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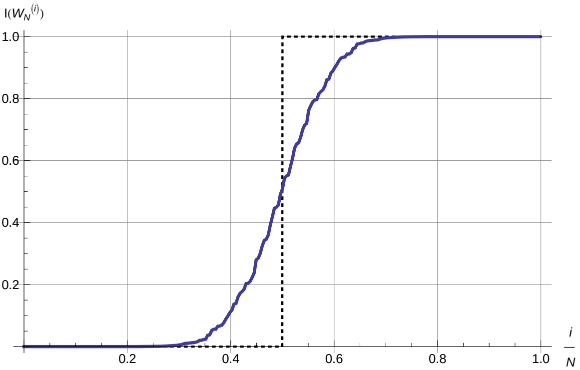

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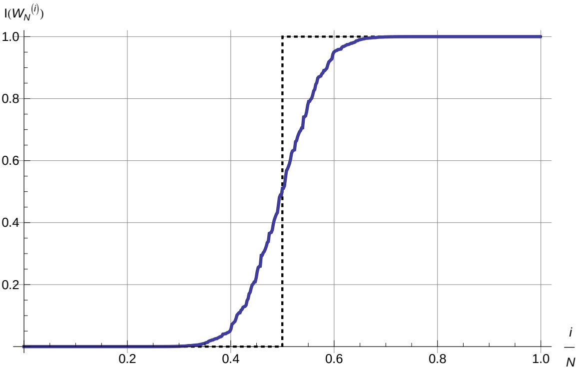

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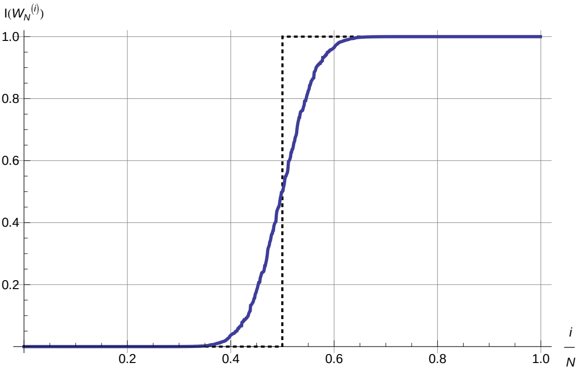

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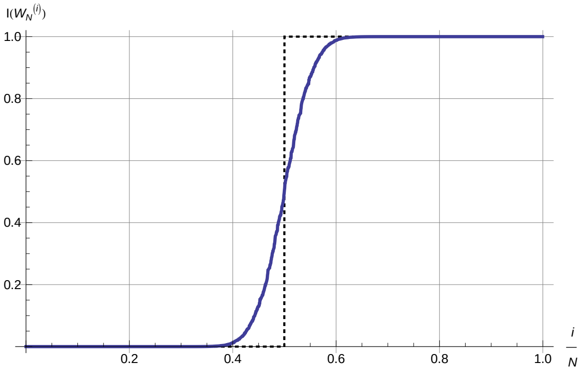

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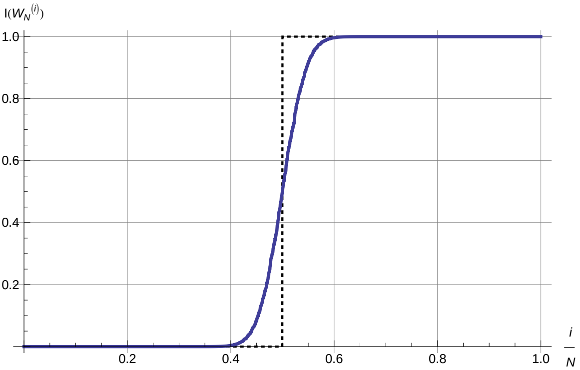

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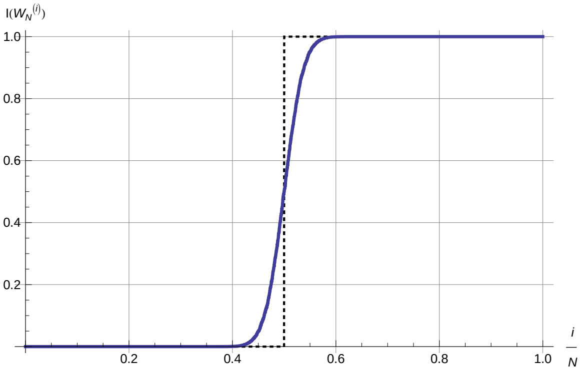
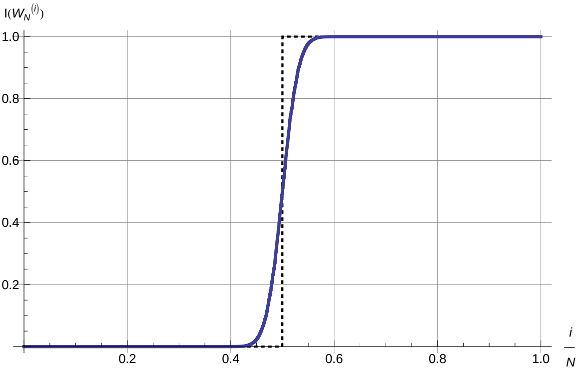


Binary polar codes: ordered bits, $N = 2^{15}$



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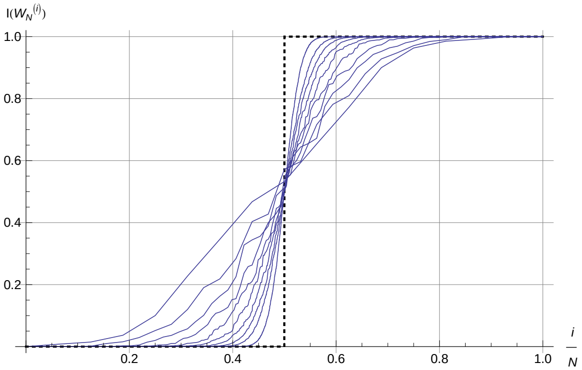
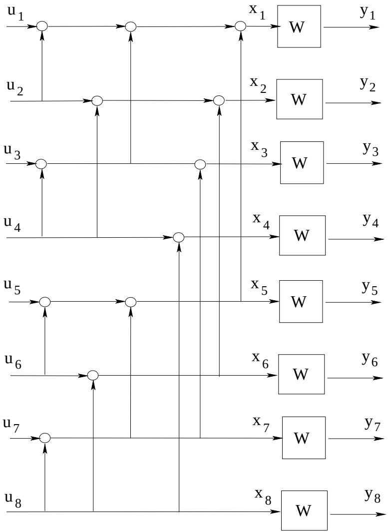


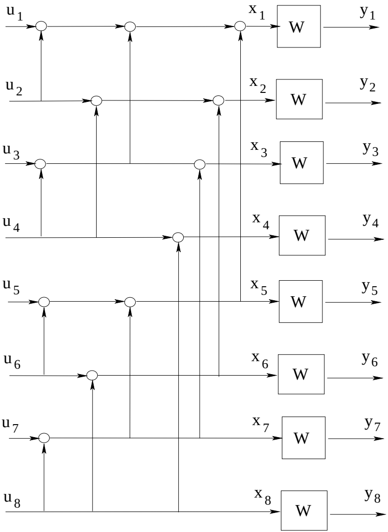
Encoding map

$C(W_i)$
0.0039
0.1211
0.1914
0.6836
0.3164
0.8086
0.8789
0.9961



Encoding map

$C(W_i)$	rank
0.0039	8
0.1211	7
0.1914	6
0.6836	4
0.3164	5
0.8086	3
0.8789	2
0.9961	1



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0.0039	8	
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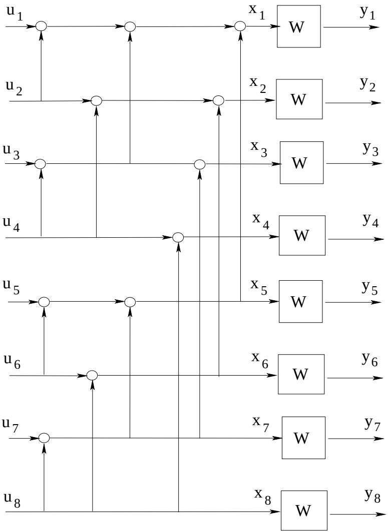
0.8789	2	
0.9961	1	data



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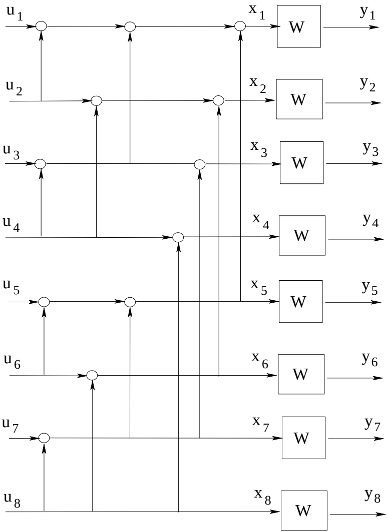

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0.0039	8	
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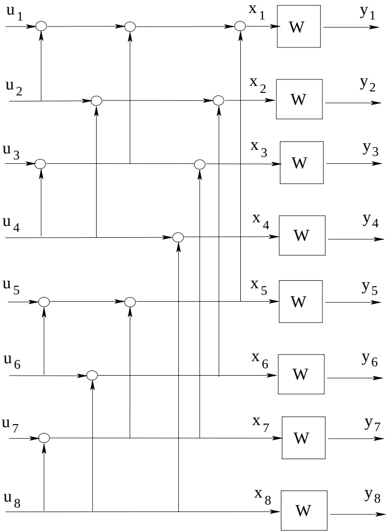

Encoding map

$C(W_i)$	rank	
0.0039	8	
0.1211	7	
0.1914	6	
0.6836	4	data
0.3164	5	
0.8086	3	data
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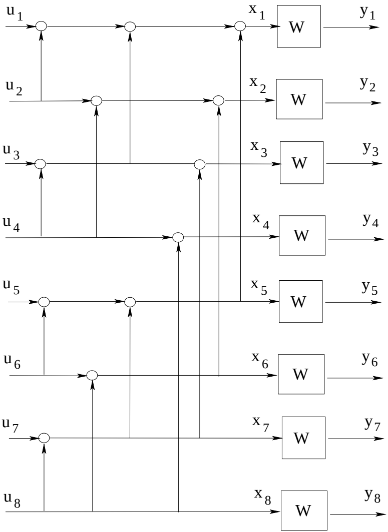
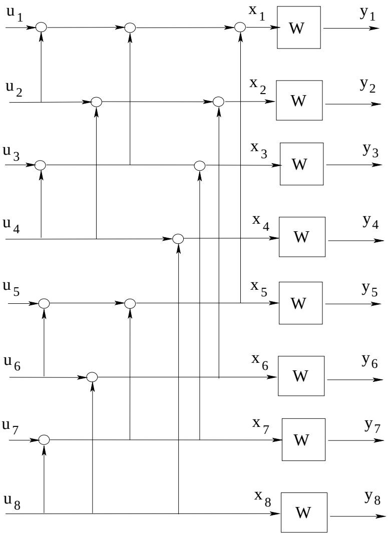
0.9961	1	data



Encoding map

$C(W_i)$	rank	
0.0039	8	frozen
0.1211	7	frozen
0.1914	6	frozen
0.6836	4	data
0.3164	5	frozen
0.8086	3	data
0.8789	2	data
0.9961	1	data



Encoding map

$C(W_i)$	rank	
0.0039	8	0
0.1211	7	0
0.1914	6	0
0.6836	4	data
0.3164	5	0
0.8086	3	data
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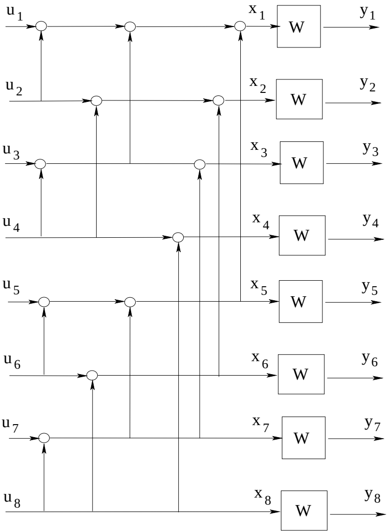

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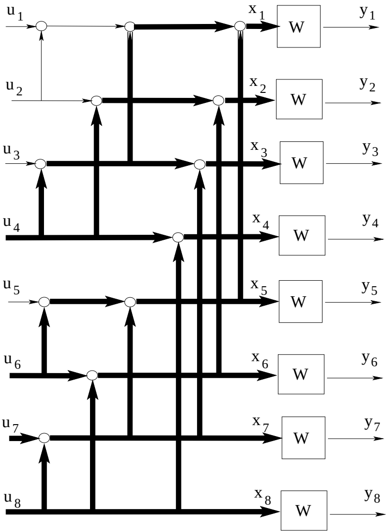

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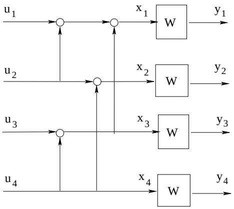
$$x_1^4 = u_1^4 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



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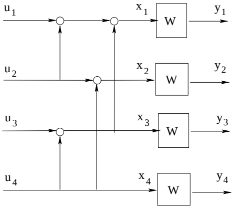

Generally $x_1^N = u_1^N H_N$, where $H_N = H_2^{\otimes n}, N = 2^n$

The set $\{1, \dots, N\}$ contains $NI(W)$ indices such that $I(W^s) \approx 1$.

Binary polar codes: Convergence

Why this works:

$$I(W^-) + I(W^+) = 2I(W)$$

After n steps we obtain 2^n values $\mathcal{I}_n = \{I^b, b \in \{+,-\}^n\}$

Binary polar codes: Convergence

Why this works:

$$I(W^-) + I(W^+) = 2I(W)$$

After n steps we obtain 2^n values $\mathcal{I}_n = \{I^b, b \in \{+,-\}^n\}$

Introduce a uniform distribution on $\mathcal{I}_n$: $P_n(I^b) = 2^{-n}$ for all b
Consider the random process $I_n, n \geq 1$.

The sequence I_n forms a bounded martingale: $E(I_{n+1}|\mathcal{F}_n) = I_n$

$$I_n \overset{\text{a.s.}}{\to} I_\infty$$

$$I_\infty \in \{0, 1\}, \; EI_\infty = I(W)$$

Binary polar codes: Convergence

For any $\epsilon > 0$

$$\lim_{n\to\infty} \frac{|\{b \in \{+,-\}^n : I(W^b) \in (\epsilon, 1-\epsilon)\}|}{2^n} = 0.$$

Decoding of polar codes

Let $A_N \subset \{1, \ldots, N\}$ be the set of bits used to transmit data

Successive cancellation (SC decoding, Arikan '09)

$$\hat{u}_i = \begin{cases} \arg\max_{z \in \{0,1\}} W(y_1^N, \hat{u}_1^{i-1} | z) & \text{if } i \in F_N^c \\ 0 & \text{if } i \in F_N. \end{cases}$$

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Rate of polarization (Arikan-Telatar '09)
The decline rate of BER is given by $O(2^{-\sqrt{N}})$

q-ary polar codes, $q = 2^r$

Arikan's kernel: $(x_1, x_2) = (u_1, u_2)H_2$ where $H_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

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$$\boxed{(x_1, x_2, \dots, x_N) = (u_1, u_2, \dots, u_N)(H_2)^{\otimes n}, \quad N = 2^n}$$

q-ary polar codes, $q = 2^r$

Arikan's kernel: $(x_1, x_2) = (u_1, u_2)H_2$ where $H_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

$$\boxed{(x_1, x_2, \dots, x_N) = (u_1, u_2, \dots, u_N)(H_2)^{\otimes n}, \quad N = 2^n}$$

$$(x_1, x_2, \dots, x_8) = (u_1, u_2, \dots, u_8) \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Polar codes for q-ary alphabets, $q = 2^r$

Let W_n be the random channel at step n,

$$Pr(W_n = W^B, B \in \{+, -\}^n) = 2^{-n}$$

$I_n = I(W_n)$ – symmetric capacity

Theorem

$I_n \to I_\infty$ a.e., where I_∞ is supported on the set $\{0, 1, \ldots, r\}$ and $EI_\infty = I(W)$.

Extremal configurations

The virtual channels converge to one of $r + 1$ possibilities:

$$\begin{array}{cccccc} 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & 1 & \dots & 1 & 1 \\ \vdots & & \vdots & & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{array}$$

Extremal configurations

Define the channel ”for the last k bits”:

$$W^{[k]}(y|u) = \frac{1}{2^{r-k}} \sum_{x \in \mathcal{X}: x_{r-k+1}^r = u} W(y|x), \qquad u \in \{0,1\}^k$$

Theorem

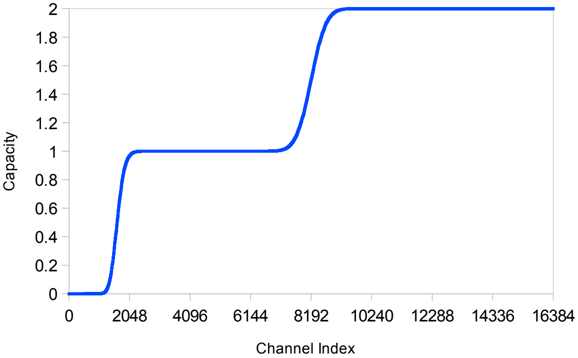
For any DMC $W : \mathcal{X} \to \mathcal{Y}$ the channels $W_N^{(i)}$ polarize to one of the $r+1$ extremal configurations. Namely, let $V_i = W_N^{(i)}$ and

$$\pi_{k,N} = \frac{|\{i \in [N] : |I(V_i) - k| < \delta \wedge |I(V_i^{[k]}) - k| < \delta\}|}{N},$$

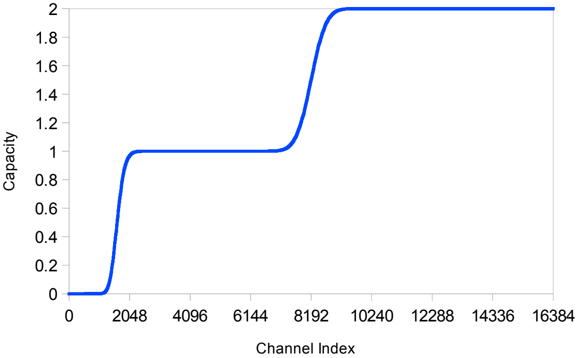
where $\delta > 0$, then $\lim_{N\to\infty} \pi_{k,N} = P(I_\infty = k)$ for all $k = 0, 1, \dots, r$. Consequently

$$\sum_{k=1}^{r} k\pi_k \to I(W).$$

Extremal configurations: Example



Extremal configurations: Example



A.B, W. Park, Polar codes for q-ary channels, $q = 2^r$, IEEE Trans, Inform. Theory, in press, arXiv:1107.4965v3

Notes for polar codes: http://www.ece.umd.edu/∼abarg/627/polar.pdf

Polar codes and RM codes

Polar codes are related to classical Reed-Muller codes

RM(m, r) a code of length $N = 2^n$, $k = \sum_{i=0}^{r} \binom{m}{i}$ data symbols, distance 2^{m-r}

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1000 0000 0000 0000
1100 0000 0000 0000
1010 0000 0000 0000
1111 0000 0000 0000
1000 1000 0000 0000
1100 1100 0000 0000
1010 1010 0000 0000
1111 1111 0000 0000
1000 0000 1000 0000
1100 0000 1100 0000
1010 0000 1010 0000
1111 0000 1111 0000
1000 1000 1000 1000
1100 1100 1100 1100
1010 1010 1010 1010
1111 1111 1111 1111

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RM(m, r) a code of length $N = 2^n$, $k = \sum_{i=0}^{r} \binom{m}{i}$ data symbols, distance 2^{m-r}

RM (0,4)	Polar
1000 0000 0000 0000	1000 0000 0000 0000
1100 0000 0000 0000	1100 0000 0000 0000
1010 0000 0000 0000	1010 0000 0000 0000
1111 0000 0000 0000	1111 0000 0000 0000
1000 1000 0000 0000	1000 1000 0000 0000
1100 1100 0000 0000	1100 1100 0000 0000
1010 1010 0000 0000	1010 1010 0000 0000
1111 1111 0000 0000	1111 1111 0000 0000
1000 0000 1000 0000	1000 0000 1000 0000
1100 0000 1100 0000	1100 0000 1100 0000
1010 0000 1010 0000	1010 0000 1010 0000
1111 0000 1111 0000	1111 0000 1111 0000
1000 1000 1000 1000	1000 1000 1000 1000
1100 1100 1100 1100	1100 1100 1100 1100
1010 1010 1010 1010	1010 1010 1010 1010
1111 1111 1111 1111	1111 1111 1111 1111

Polar codes and RM codes

Polar codes are related to classical Reed-Muller codes

RM(m, r) a code of length $N = 2^n$, $k = \sum_{i=0}^{r} \binom{m}{i}$ data symbols, distance 2^{m-r}

RM (1,4)	Polar
1000 0000 0000 0000	1000 0000 0000 0000
1100 0000 0000 0000	1100 0000 0000 0000
1010 0000 0000 0000	1010 0000 0000 0000
1111 0000 0000 0000	1111 0000 0000 0000
1000 1000 0000 0000	1000 1000 0000 0000
1100 1100 0000 0000	1100 1100 0000 0000
1010 1010 0000 0000	1010 1010 0000 0000
1111 1111 0000 0000	1111 1111 0000 0000
1000 0000 1000 0000	1000 0000 1000 0000
1100 0000 1100 0000	1100 0000 1100 0000
1010 0000 1010 0000	1010 0000 1010 0000
1111 0000 1111 0000	1111 0000 1111 0000
1000 1000 1000 1000	1000 1000 1000 1000
1100 1100 1100 1100	1100 1100 1100 1100
1010 1010 1010 1010	1010 1010 1010 1010
1111 1111 1111 1111	1111 1111 1111 1111

Polar codes and RM codes

Polar codes are related to classical Reed-Muller codes

RM(m, r) a code of length $N = 2^n$, $k = \sum_{i=0}^{r} \binom{m}{i}$ data symbols, distance 2^{m-r}

RM (2,4)	Polar
1000 0000 0000 0000	1000 0000 0000 0000
1100 0000 0000 0000	1100 0000 0000 0000
1010 0000 0000 0000	1010 0000 0000 0000
1111 0000 0000 0000	1111 0000 0000 0000
1000 1000 0000 0000	1000 1000 0000 0000
1100 1100 0000 0000	1100 1100 0000 0000
1010 1010 0000 0000	1010 1010 0000 0000
1111 1111 0000 0000	1111 1111 0000 0000
1000 0000 1000 0000	1000 0000 1000 0000
1100 0000 1100 0000	1100 0000 1100 0000
1010 0000 1010 0000	1010 0000 1010 0000
1111 0000 1111 0000	1111 0000 1111 0000
1000 1000 1000 1000	1000 1000 1000 1000
1100 1100 1100 1100	1100 1100 1100 1100
1010 1010 1010 1010	1010 1010 1010 1010
1111 1111 1111 1111	1111 1111 1111 1111

Decoding of polar codes and RM codes

Goal: Fill the void for moderate block length: $200 \leq N \leq 2000$

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Problem: Relatively slow convergence

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Problem: Relatively slow convergence

Enhancements of Decoding Algorithms:
List decoding
Gradient-like decoding
Easier logic by quantizing SC decoding

A.B. and I. Dumer (UC Riverside), work in progress

I. Dumer, papers on decoding RM codes, *IEEE Transactions on Information Theory*, 2006,2008.

List decoding of polar codes

SC decoding:

$$\hat{u}_i = (\arg\max_{z\in\{0,1\}} W(y_1^N, \hat{u}_1^{i-1}|z)) \cdot I_{\{i\in F_N^c\}}$$

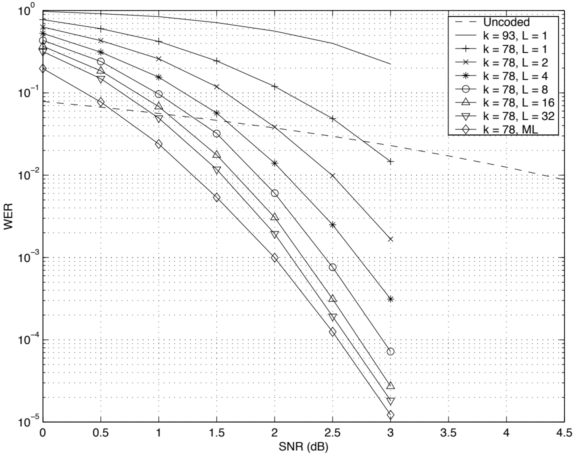
List decoding of polar codes

SC decoding:

$$\hat{u}_i = (\arg\max_{z\in\{0,1\}} W(y_1^N, \hat{u}_1^{i-1}|z)) \cdot I_{\{i\in F_N^c\}}$$

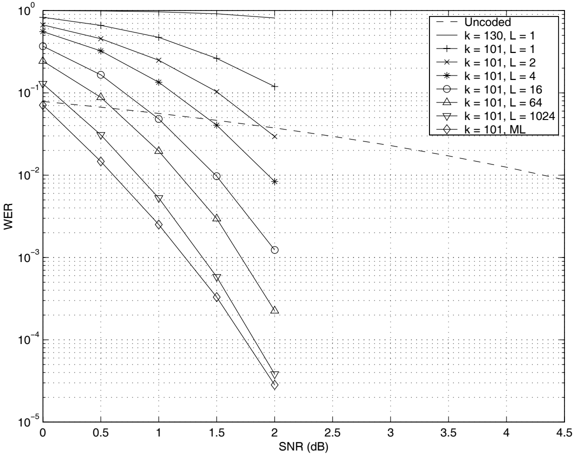
Keep $L = 2^s$ most probable bit sequences $(i_1, \ldots, i_s)$, start pruning the list after that.

List decoding of polar codes



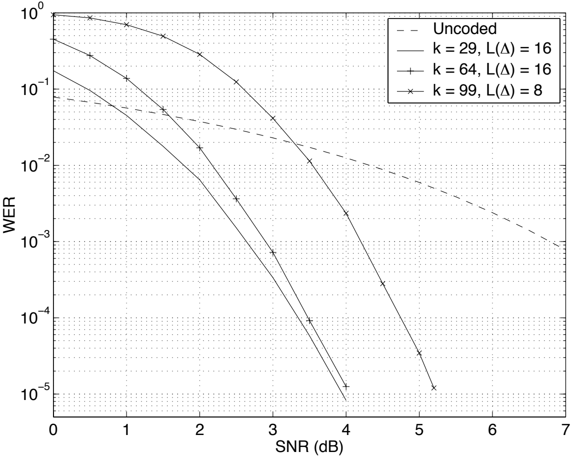
(256,78) Polar Code $\subset$ (256,93) RM(8,3) code

List decoding of polar codes



(512,101) Polar Code $\subset$ (512,130) RM(9,3) code

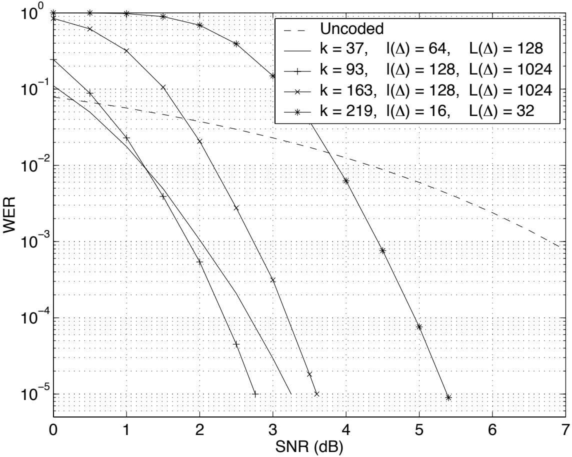
List decoding of polar codes



RM code of length $N = 128$

$L(\Delta)$=list size required for BER withing 0.25 dB of ML performance

List decoding of polar codes



RM codes of length $N = 256$

$L(\Delta)$=list size required for BER withing 0.25 dB of ML performance

List decoding of polar codes

List decoding with CRC achieves state-of-the-art performance

Quantization

Let $\rho_i(W) \triangleq L_N^i(y_1^N, \hat{u}_1^{i-1}) = \log \frac{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|0)}{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|1)}$

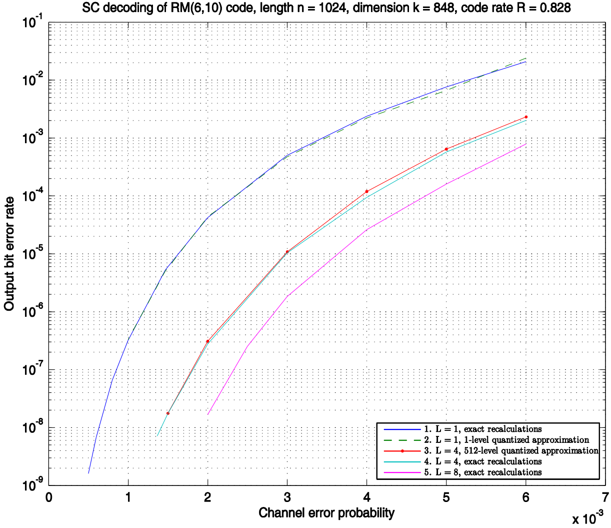
Recurstion step:

$$\rho(W^-) = \log \frac{e^{\rho' + \rho''} + 1}{e^{\rho'} + e^{\rho''}}, \quad \rho(W^+) = \rho' + x \cdot \rho''$$

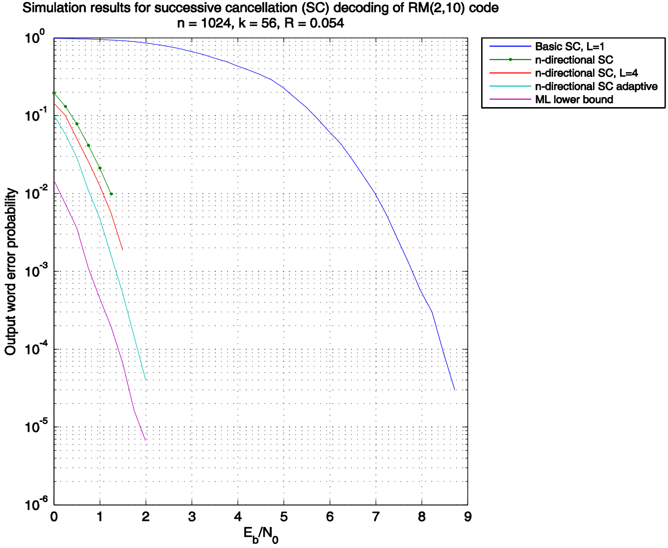
where x is the value of the decoded symbol.

Table-based approximation with only small loss of accuracy

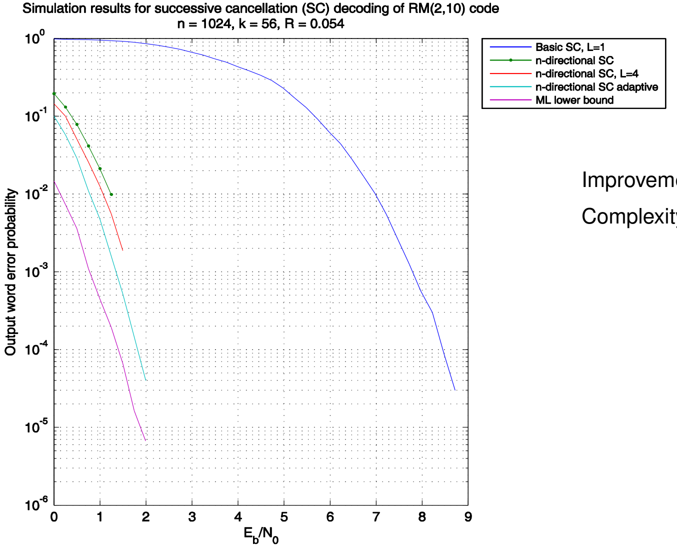
Quantization



Gradient-like decoder



Gradient-like decoder



Improvement for $N = 2^{10} - N = 2^{12}$

Complexity increases by a factor of N